Density functional theory studies of hetero-Diels–Alder reactions[†],[‡]

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Transition structures for hetero-Diels–Alder reactions involving the heteroatoms O, S and N in dienes as well as in dienophiles have been determined at the MP2 (MP2/6-31G*//MP2/6-31G*) and hybrid DFT (B3LYP/6-31G*//B3LYP/6-31G*) levels of theory. The transition structures are predicted to be relatively early, concerted and asynchronous in all cases, with the DFT transition structures being more asynchronous than the MP2 ones. The reactions of butadiene with formaldehyde, thioformaldehyde and formaldimine proceed by diene HOMO–dienophile LUMO interactions whereas those of ethylene with acrolein, 1-thiabutadiene and 1-azabutadiene proceed by reverse electron demand interactions. In all the hetero-Diels–Alder reactions, the C–C bond is more fully formed than the heteroatom–carbon bond in the transition structure with the exception of the reaction between formaldimine and butadiene for which C–N bond formation is ahead of C–C bond making. All the reactions are highly exothermic. The reactions are facilitated by heteroatoms in both the diene and dienophile, and bond formation between two heteroatoms is disfavored. The reaction of formaldimine shows an *endo* preference whereas that of 1-azabutadiene leads to an *exo* preference of the imino hydrogen; these preferences of *ca.* 4 kcal mol^{-1} are caused mostly by inter-hydrogen steric effects in the transition structures. We conclude that the DFT calculations provide an economical way of accounting for electron correlation effects at nearly the same level as the MP2 ones in the investigations of hetero-Diels–Alder reactions.

Hetero-Diels–Alder reactions are an important method for synthesizing various types of heterocyclic compounds.[1] The heteroatoms commonly involved in these reactions are N, O and S, and they can be either in a diene and/or in a dienophile. Although the reactions are generally believed to proceed through a concerted, asynchronous transition state (TS), a stepwise pathway is also thought to be a possibility. There have been a number of reports of mechanistic[2] and theoretical studies[3,4] on hetero-Diels–Alder reactions.

Houk and coworkers have published several theoretical studies on the transition structures of Diels–Alder reactions.[4a–g] The most thorough high level MO studies were on the mechanism of the prototype Diels–Alder reaction of butadiene and ethylene.[4a] In this work they applied density functional theory at the Becke3LYP/6-31G* level,[5] and found that the lowest energy stepwise pathway has a free energy of activation 7.7 kcal mol^{-1} above that of the concerted path. An important conclusion of their work was that the QCISD(T)/6-31G*//CASSCF/6-31G* calculation[6] is the only level thus far which yields results comparable to the hybrid DFT (B3LYP/6-31G*) method that they applied. In another paper, Houk and coworkers[3d] studied transition structures for the reactions between butadiene and heterodienophiles with heteroatoms O, S, N and B at relatively low theoretical levels: RHF/3-21G//RHF/3-21G, RHF/6-31G*//RHF/3-21G$^{(*)}$ and MP2/6-31G*//RHF/3-21G*.[6] Due to the low theoretical levels they applied, some of their results appeared unreasonable. For example, in the transition structure for the reaction between formaldehyde ($H_2C{=}O$) and butadiene, the forming O–C bond was shorter by 0.135 Å than the forming C–C bond. This does not conform to frontier orbital theory, according to which the C–C bond should be more fully formed than the O–C bond since the formaldehyde carbon has a larger LUMO coefficient than the oxygen.[7] Recently, Whiting and Windsor[3g] reported the results of *ab initio* studies on the reactions between butadiene and various substituted azadienophiles at the MP2/6-31G*//HF/3-21G* level. Their results indicated that all the reactions proceed by a concerted asynchronous cycloaddition through diene HOMO–dienophile LUMO interactions.

In this work, we carried out theoretical studies on 14 hetero-Diels–Alder reactions with heteroatoms O, S and N in dienes (acrolein, 1-thiabutadiene and 1-azabutadiene) and/or in dienophiles (formaldehyde, thioformaldehyde and formaldimine) at the MP2/6-31G*//MP2/6-31G*[6] and Becke3LYP/6-31G*//Becke3LYP/6-31G*[5] levels of theory. We aimed to examine the transition structures using the most economical and satisfactory method thus far available (B3LYP/6-31G*)[4a,k] and to predict the regioselectivity, stereoselectivity and reactivity of the hetero-Diels–Alder reactions.

Experimentally, hetero-Diels–Alder reactions are commonly carried out (often in solution) using heterodienes and/or heterodienophiles with activating substituents[1] in contrast to the gas-phase reactions between the simple unsubstituted reactants investigated in this work. This means that directly comparable experimental data with our theoretical results will be very scarce.

Calculations

Calculations were carried out with the Gaussian 92 and 94 series of programs.[8] Geometries of reactants, transition structures and products were fully optimized at the MP2/6-31G*[6] and Becke3LYP/6-31G* levels[5] of theory and energies are

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[†] *Non-SI units employed*: 1 kcal ≈ 4.18 kJ; 1 a.u. = 1 hartree ≈ 2.63×10^3 kJ mol^{-1}, 1 atm ≈ 1 bar = 10^5 Pa.

[‡] Supplementary material available: tables of calculated bond lengths and imaginary vibrational frequencies. For direct electronic access see http://www.rsc.org/suppdata/nj/1999/707/, otherwise available from BLDSC (No. SUP 57559, 4 pp.) or the RSC Library. See Instructions for Authors, 1999, Issue 1 (http://www.rsc.org/njc).

reported at the MP2/6-31G*//MP2/6-31G* and B3LYP/6-31G*//B3LYP/6-31G* levels, which will be hereafter referred to as the MP2 and DFT results. Frequency calculations were performed at the MP2/6-31G* and B3LYP/6-31G* levels, respectively, to characterize the nature of stationary points, including the transition structures (which has only one imaginary frequency) and to determine zero-point energies and entropies.

Activation ($\Delta E^{\neq}$) and reaction energies (ΔE°) are calculated starting from the *trans* forms of the dienes and both *exo* and *endo* transition structures were located for the reactions involving an imino nitrogen atom, that is formaldimine and/or 1-azabutadiene.

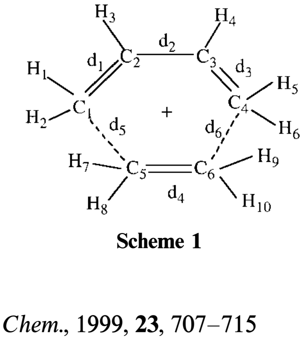
Results and discussion

Although the mechanism of the hetero-Diels–Alder reaction is generally assumed to be a concerted, asynchronous cycloaddition, a stepwise, radical-mediated mechanism can also occur.[2e,f,3g] In the present work, however, we have limited our investigation to the *restricted* B3LYP/6-31G* calculations of the *concerted pathway* and have not considered the diradical stepwise mechanism separately by carrying out unrestricted B3LYP/6-31G* optimization of the open-shell pathway.

Altogether 15 hetero-Diels–Alder reactions were studied in the present work, including the prototype reaction of butadiene and ethylene (reaction 1). A concerted but asynchronous transition structure was found for all of them. We have repeated the DFT calculation on reaction 1 in order to check the methodology, and we found that our DFT results for this reaction agreed with those of Houk and coworkers, and others.[4a,9,10] The MP2 results in general show the same trends, as far as the regioselectivity, stereoselectivity and asynchronicity (the relative extents of the two bond formation processes in the transition structure) are concerned, as the DFT calculations (*vide infra*). With few exceptions, however, the MP2 correlation energy correction is found to overestimate the effect of electron correlation in the transition structure and lowered the activation energy too much. Similarly, the product energies are also lowered too much so that the MP2 reaction energies are more negative than the corresponding DFT results. Consequently, the MP2 transition structure tends to be at a relatively earlier position than the DFT transition structure (*e.g.*, for reaction 1, the forming C–C bond length is 2.285 and 2.272 Å, respectively, in the transition structures) along the reaction coordinate. Furthermore, the DFT transition structures are significantly more asynchronous than the MP2 structures.

To aid our discussion below, each atom is numbered with an all-carbon structure as shown in Scheme 1. In this work, we adopt the convention of placing a heteroatom at C_1 in the diene and at C_6 in the dienophile.

The MP2 and DFT bond lengths between heavy atoms in the transition structures are summarized in Table 1, and the MP2 and DFT reactant energies in Table 2. The activation ($\Delta E^{\neq}$) and reaction energies and entropies (ΔE° and $\Delta S^{\neq}$) with respect to reactants (*cis* form of diene + dienophile) are collected in Table 3. In Table 4 we give the frontier molecular orbital levels for the dienes and dienophiles.



Scheme 1

Reactions involving dienophile and/or diene with a terminal oxygen heteroatom

Reaction of butadiene and formaldehyde: reaction 2 (CH_2=CH–CH=CH_2 + CH_2=O). The DFT transition structure for this reaction is substantially asynchronous; the forming C–C (d_5) bond length is shorter by 0.148 Å than the C–O (d_6) bond (2.144 Å, see Table 1). These two bonds are much shorter (by more than 0.2 Å) than the corresponding bonds in the synchronous transition structure of butadiene plus ethylene (reaction 1). The percentage bond formation calculated, based on bond order changes, is 46 and 30% for d_5 and d_6, respectively. The MP2 results show, however, considerably less asynchronous structure, the d_5 bond being shorter than d_6 by only 0.096 Å. Nevertheless, both the DFT and MP2 results predict the same trend ($d_5 < d_6$), which is in line with what is expected based on frontier molecular orbital (FMO) theory. Experimentally, in the additions of glyoxylates (RCH=O with R = CO_2R') with unsymmetrical dienes, the regiochemistry is controlled by the normal electron demand interactions in agreement with FMO theory prediction.[12] It has been suggested that hetero-Diels–Alder reactions of carbonyl dienophiles proceed in general as predicted by FMO theory.[5a] The FMO levels in Table 4 show that this reaction is a normal electron demand type reaction (diene HOMO–dienophile LUMO; $\Delta\varepsilon_{\mathrm{FMO}}$ for the normal *vs.* reverse electron demand is 0.458 *vs.* 0.657 hartree). Natural population analyses (NPA)[13] show that there is a net electronic charge flow from the diene to dienophile. This means that the formaldehyde carbon has a larger LUMO coefficient (the MP2 $2p_z$ and $3p_z$ coefficients are 0.405 and 0.711, respectively) than the oxygen ($2p_z$ and $3p_z$ coefficients are −0.380 and −0.525) so that d_5 should be more fully formed than d_6 in the transition structure. This result of the asynchronous TS with d_5 shorter than d_6 is in contrast to the transition structure predicted at the 3-21G level[3d] where d_5 (2.133 Å) is longer than d_6 (1.998 Å), which is opposite to the trend expected based on FMO theory. This shows that proper accounting of electron correlation effects is essential in transition structure prediction, especially when a heteroatom is involved. A MINDO/3 calculation[3a] predicted a correct asynchronous transition structure with longer d_6 than d_5, which suggests partial account of the electron correlation effects in the process of incorporating empirical parameters in the semiempirical method. The MP2 activation and reaction energies, $\Delta E^{\neq}$ and ΔE°, for reaction 2 (Table 3) are higher by *ca.* 4 and 20 kcal mol^{-1}, respectively, than those for reaction 1, whereas the corresponding differences of the DFT values are *ca.* 0 and 16 kcal mol^{-1}. A greater increase in the MP2 reaction energy compared to the DFT values of *ca.* 4 kcal mol^{-1} is reflected in a similar difference in the activation energies. It is interesting to note that the DFT activation energies for reactions 1 and 2 and the MP2 activation energy for reaction 2 are practically the same (∼21 kcal mol^{-1}) despite the fact that there is a more than 15 kcal mol^{-1} difference in the reaction energies between reactions 1 and 2. This may be due to the relatively low degree of progress in d_6 (O–C) bond making in the TS; the DFT and MP2 results give ∼30% of d_6 bond making in both reactions 1 and 2. The reaction of formaldehyde (reaction 2) is, however, entropically unfavorable compared to that of ethylene (reaction 1) so that the Gibbs free energy of activation, $\Delta G^{\neq}$, is slightly higher, by 5.2 (MP2) and 0.8 (DFT) kcal mol^{-1}.

Although there are many examples of uncatalyzed Diels–Alder reactions using aldehydes and ketones activated by electron-acceptor substituents, formaldehyde fails to react with simple 1,3-dienes including 1,3-butadiene.[14] A wide range of aldehydes and ketones react readily with certain reactive 1,3-dienes only at elevated temperatures, under high pressures or in the presence of a Lewis acid catalyst.[15]

Table 1 Bond lengths (Å) and percentage bond formation[a] (given in parentheses) for transition structures

Reaction	No.	Method	d_1	d_2	d_3	d_4	d_5	d_6
CH_2=CH–CH=CH_2 + CH_2=CH_2	1	MP2	1.380 (28)	1.412 (40)	1.380 (28)	1.382 (30)	2.286 (28)	2.285 (28)
		(HF)	(1.377)	(1.393)	(1.377)	(1.383)	(2.202)	(2.202)
		DFT	1.383 (31)	1.407 (42)	1.383 (31)	1.382 (34)	2.272 (29)	2.272 (29)
CH_2=CH–CH=CH_2 + CH_2=O	2	MP2	1.393 (37)	1.405 (45)	1.376 (25)	1.282 (37)	2.003 (45)	2.099 (32)
		(HF)	(1.392)	(1.388)	(1.377)	(1.258)	(2.002)	(2.038)
		DFT	1.400 (43)	1.403 (44)	1.379 (29)	1.274 (39)	1.996 (46)	2.144 (30)
O=CH–CH=CH_2 + CH_2=CH_2	3	MP2	1.272 (36)	1.409 (47)	1.391 (37)	1.388 (35)	2.148 (31)	2.032 (43)
		(HF)	(1.246)	(1.386)	(1.400)	(1.386)	(2.112)	(1.998)
		DFT	1.263 (36)	1.406 (47)	1.400 (43)	1.390 (37)	2.186 (29)	2.039 (43)
O=CH–CH=CH_2 + CH_2=O	4	MP2	1.280 (42)	1.401 (52)	1.389 (36)	1.268 (32)	1.943 (42)	1.896 (46)
		(HF)	(1.253)	(1.382)	(1.401)	(1.242)	(1.940)	(1.845)
		DFT	1.273 (43)	1.400 (51)	1.396 (41)	1.260 (35)	1.950 (42)	1.923 (44)
CH_2=CH–CH=CH_2 + CH_2=S	5	MP2	1.370 (21)	1.423 (32)	1.372 (22)	1.654 (24)	2.393 (23)	2.532 (30)
		(HF)	(1.377)	(1.403)	(1.362)	(1.673)	(2.109)	(2.625)
		DFT	1.375 (26)	1.420 (32)	1.369 (22)	1.669 (30)	2.293 (28)	2.652 (26)
S=CH–CH=CH_2 + CH_2=CH_2	6	MP2	1.659 (27)	1.417 (36)	1.374 (23)	1.381 (30)	2.567 (29)	2.320 (27)
		(HF)	(1.667)	(1.384)	(1.388)	(1.376)	(2.567)	(2.135)
		DFT	1.676 (32)	1.406 (40)	1.384 (30)	1.840 (29)	2.626 (27)	2.257 (30)
S=CH–CH=CH_2 + CH_2=S	7	MP2	1.643 (13)	1.436 (22)	1.364 (15)	1.634 (12)	2.754 (21)	2.606 (27)
		(HF)	(1.666)	(1.392)	(1.376)	(1.661)	(2.460)	(2.487)
		DFT	1.657 (16)	1.434 (19)	1.361 (13)	1.638 (12)	2.848 (18)	2.802 (20)
CH_2=CH–CH=CH_2 + CH_2=NH (*exo*)	8	MP2	1.388 (33)	1.408 (43)	1.393 (27)	1.333 (34)	2.158 (35)	2.159 (32)
		(HF)	(1.383)	(1.390)	(1.385)	(1.314)	(2.174)	(2.020)
		DFT	1.390 (36)	1.404 (44)	1.398 (34)	1.326 (35)	2.215 (32)	2.094 (35)
CH_2=CH–CH=CH_2 + CH_2=NH (*endo*)	9	MP2	1.379 (27)	1.412 (40)	1.386 (32)	1.328 (31)	2.324 (27)	2.038 (39)
		(HF)	(1.374)	(1.394)	(1.393)	(1.306)	(2.321)	(1.912)
		DFT	1.378 (28)	1.410 (39)	1.397 (40)	1.317 (30)	2.432 (23)	1.953 (45)
NH=CH–CH=CH_2 (*exo*) + CH_2=CH_2	10	MP2	1.325 (37)	1.413 (43)	1.382 (30)	1.383 (31)	2.209 (29)	2.159 (35)
		(HF)	(1.305)	(1.393)	(1.385)	(1.384)	(2.129)	(2.092)
		DFT	1.319 (38)	1.410 (44)	1.387 (35)	1.386 (34)	2.196 (29)	2.163 (35)
NH=CH–CH=CH_2 (*endo*) + CH_2=CH_2	11	MP2	1.331 (42)	1.408 (48)	1.393 (37)	1.392 (36)	2.278 (26)	2.079 (40)
		(HF)	(1.313)	(1.389)	(1.391)	(1.390)	(2.170)	(2.066)
		DFT	1.327 (45)	1.404 (50)	1.398 (42)	1.396 (40)	2.284 (25)	2.066 (42)
NH=CH–CH=CH_2 + CH_2=NH (*exo*,*exo*)	12	MP2	1.328 (39)	1.409 (46)	1.385 (32)	1.319 (27)	2.127 (33)	2.007 (41)
		(HF)	(1.291)	(1.409)	(1.426)	(1.253)	(3.019)	(1.687)
		DFT	1.322 (40)	1.409 (45)	1.394 (39)	1.309 (27)	2.205 (29)	1.982 (43)
NH=CH–CH=CH_2 + CH_2=NH (*exo*,*endo*)	13	MP2	1.319 (30)	1.417 (40)	1.385 (32)	1.312 (21)	2.245 (27)	1.971 (43)
		(HF)	(1.297)	(1.403)	(1.405)	(1.278)	(2.391)	(1.808)
		DFT	1.313 (32)	1.417 (39)	1.393 (38)	1.302 (22)	2.317 (24)	1.962 (44)
NH=CH–CH=CH_2 + CH_2=NH (*endo*,*exo*)	14	MP2	1.339 (49)	1.403 (52)	1.395 (38)	1.326 (32)	2.194 (30)	1.930 (47)
		(HF)	(1.317)	(1.382)	(1.469)	(1.276)	(2.599)	(1.539)
		DFT	1.329 (46)	1.404 (50)	1.413 (50)	1.310 (28)	2.400 (21)	1.833 (55)
NH=CH–CH=CH_2 + CH_2=NH (*endo*,*endo*)	15	MP2	1.322 (34)	1.416 (43)	1.398 (40)	1.320 (22)	2.424 (24)	1.859 (53)
		(HF)	(1.297)	(1.406)	(1.422)	(1.276)	(2.661)	(1.704)
		DFT	1.314 (33)	1.418 (41)	1.411 (49)	1.306 (24)	2.613 (20)	1.810 (57)

[a] Percentage bond formation based on bond order changes calculated by $\% \Delta n^{\neq} = \exp(-r^{\neq}/a) - \exp(-r_{\mathbf{R}}/a)/\exp(-r_{\mathbf{P}}/a) - \exp(-r_{\mathbf{R}}/a)$ where $r_{\mathbf{R}}$, $r^{\neq}$ and $r_{\mathbf{P}}$ are the bond lengths in the reactant, TS and product, respectively, and a is a constant: $a = 0.6$ for partial bonds but $a = 0.3$ for covalent bonds.

Reaction of acrolein and ethylene: reaction 3 (O=CH–CH=CH_2 + CH_2=CH_2). Table 1 shows that the d_6 (C–C) bond is more fully formed (43%) than the d_5 (O–C) bond (29%). The transition structure is formed somewhat earlier on the reaction coordinate than in reaction 2. The d_6 bond is shorter by 0.147 Å in the DFT results while it is 0.116 Å shorter in the MP2 ones, indicating again that the DFT transition structure is more asynchronous than the MP2 structure. The FMO levels in Table 4 show that this reaction is a reverse electron demand type (diene LUMO–dienophile HOMO; $\Delta\varepsilon_{\text{FMO}}$ for the normal *vs.* reverse electron demand is 0.565 *vs.* 0.456 hartree). The NPA charges indicated that there is a net electronic charge flow from the dienophile ($\Delta q^{\neq}$ for C_5 and C_6 are 0.117 and 0.012) to diene ($\Delta q^{\neq}$ for O_1 and C_4 are −0.011 and −0.033). The oxygen heteroatom in the diene lowers the HOMO more than the LUMO level, and also leads to a larger MP2 LUMO ($2p_z$ and $3p_z$ are 0.270, 0.342 and 0.312, 0.542 for O_1 and C_4, respectively) as well as HOMO coefficient of C_4 ($2p_z$ and $3p_z$ are −0.244, −0.197 and 0.339, 0.264 for O_1 and C_4, respectively). The strongly electronegative terminal oxygen atom (O_1) apparently attracts π electrons inductively so that the FMO levels are lowered and π electrons on the entire diene, including those on O_1, are depleted, leading to smaller coefficients of O_1 and larger coefficients of C_4 both in the HOMO and LUMO.

Table 2 Reactant energies in a.u. (corrected for zero-point energy)

Compound	MP2/6-31G*	B3LYP/6-31G*
trans-Butadiene	−155.33629	−155.90666
cis-Butadiene	−155.33201	−155.90116
H_2CCH_2	−78.23299	−78.53624
H_2CO	−114.14046	−114.47365
H_2CS	−436.72964	−437.43744
H_2CNH	−94.27455	−94.58717
cis-$OCHCHCH_2$	−191.24694	−191.84764
cis-$SCHCHCH_2$	−513.83503	−514.81011
cis-$NHCHCHCH_2$ (*exo*)	−171.37743	−171.95517
cis-$NHCHCHCH_2$ (*endo*)	−171.37633	−171.95612
trans-$OCHCHCH_2$	−191.24935	−191.85032
trans-$SCHCHCH_2$	−513.83896	−514.81453
trans-$NHCHCHCH_2$ (*exo*)	−171.38131	−171.96049
trans-$NHCHCHCH_2$ (*endo*)	−171.37997	−171.95969

The activation barrier ($\Delta E^{\neq}$) is higher than that for reaction 2 by 2.8 (DFT) and 2.0 (MP2) kcal mol^{-1}, but the exothermicity ($-\Delta E^{\circ}$) is nearly the same for reactions 2 and 3. This

Table 3 Activation energies ($\Delta E^{\neq}$), reaction energies (ΔE°) and reaction free energies ($\Delta G^{\neq}$) in kcal mol^{-1} and activation entropies ($\Delta S^{\neq}$) in e.u. with respect to reactants (dienes in *cis* form)

	MP2[a]				DFT[b]			
Reaction	$\Delta E^{\neq}$	ΔE°	$\Delta S^{\neq}$	$\Delta G^{\neq}$	$\Delta E^{\neq}$	ΔE°	$\Delta S^{\neq}$	$\Delta G^{\neq}$
1	17.6 [29.8][c]	−48.0	−42.3	31.2	21.4 [24.8][d] (27.5)[e]	−40.1 [−36.7][d]	−42.7	35.2
2	21.5 [20.9][f]	−27.1	−44.1	36.4	21.2 [24.7]	−24.3 [−20.9]	−44.1	36.0
3	23.5 [34.0][g]	−27.3	−43.6	37.8	24.0 [25.7]	−23.4 [−21.7]	−42.9	37.9
4	22.6	−10.4	−45.6	38.4	18.0 [19.7]	−10.6 [−8.91]	−44.8	33.2
5	4.1 [3.8][f]	−44.5	−41.9	17.7	6.6 [10.1]	−38.0 [−34.6]	−41.9	20.4
6	8.7	−42.8	−43.3	22.6	11.5 [14.3]	−35.5 [−36.0]	−41.9	25.0
7	−0.4	−36.8	−42.8	13.4	−0.1 [2.7]	−31.3 [−28.5]	−39.8	12.7
8	20.4	−34.8	−45.8	35.2	22.9 [26.3] [24.7][h]	−29.4 [−26.0]	−45.9	37.8
9	16.4	−34.8	−45.7	31.2	19.1 [22.5] [20.8][h]	−29.4 [−26.0]	−45.7	34.0
10	19.0	−37.7	−45.0	33.5	21.8 [24.5]	−32.7 [−29.9]	−43.5	35.8
11	22.6	−44.5	−43.2	36.6	26.3 [29.1]	−33.3 [−30.4]	−43.6	40.5
12	19.6	−26.9	−48.2	35.4	18.9 [21.7]	−23.8 [−21.1]	−46.0	33.9
13	13.9	−26.7	−47.3	29.4	14.0 [16.8]	−23.7 [−20.9]	−46.0	29.0
14	25.5	−27.4	−46.7	40.9	25.1 [27.9]	−24.4 [−21.6]	−46.0	40.1
15	19.0	−25.9	−46.3	34.2	19.4 [22.2]	−24.3 [−21.4]	−45.7	34.3

[a] MP2/6-31G*//MP2/6-31G*. Corrected for zero-point energies. [b] Becke3LYP/6-31G*//Becke3LYP/6-31G*. Corrected for zero-point energies. [c] At the MP4SDQ/6-31G*//MP4SDQ/6-31G* level with zero-point energy corrections at the MP4SDQ/6-31G* level. The value starting from the *trans* form of butadiene is 32.4 kcal mol^{-1}. [d] Value starting from the *trans* form of butadiene. [e] Experimental value.[4,10] [f] At the MP2/6-31G*//HF/3-21G$^{(*)}$ level. [g] At the MP4SDQ/6-31G*//MP4SDQ/6-31G* level with zero-point energy corrections at the MP2/6-31G* level. The value starting from the *trans* form of acrolein is 35.4 kcal mol^{-1}. [h] At the MP4SDTQ/6-31G*//MP2/6-31G* level with zero-point energy corrections at the HF/6-31G* level.

seems to reflect a lower degree of bond formation in the TS (earlier TS) for reaction 3 than for reaction 2 (*vide supra*). The MP4SDQ/6-31G* $\Delta E^{\neq}$ value in Table 3 indicates that the DFT result is marginally better than the MP2 value.

Experimentally, it has been shown that intramolecular Diels–Alder reactions involving the acrolein/ethene system are very likely to proceed in a concerted fashion without the intermediacy of diradicaloids.[2e] The TS has been shown to be asynchronous.[2f] It is also known that 1-oxabutadiene systems are electron-deficient and hence react by reverse electron demand interactions even with simple olefinic and acetylenic dienophiles.[16] To promote the thermal reactions of 1-oxabutadienes the reactions are carried out at high temperature (150–250 °C) or under mild thermal conditions

Table 4 Frontier molecular orbital levels calculated at the HF/6-31G*//MP2/6-31G* level (ε in a.u.)

	Normal electron demand			Reverse electron demand		
Reaction	Diene HOMO	Dienophile LUMO	$\Delta\varepsilon_{\mathrm{FMO}}$	Diene LUMO	Dienophile HOMO	$\Delta\varepsilon_{\mathrm{FMO}}$
1	−0.3243	0.1791	0.5034	0.1322	−0.3701	0.5023
2	−0.3243	0.1340	0.4583	0.1322	−0.5250	0.6573
3	−0.3861	0.1791	0.5652	0.0858	−0.3701	0.4558
4	−0.3243	0.1340	0.5200	0.0858	−0.5250	0.6108
5	−0.3487	0.0535	0.3778	0.1322	−0.4143	0.5466
6	−0.3487	0.1791	0.5278	0.0346	−0.3701	0.4047
7	−0.3243	0.0535	0.4021	0.1322	−0.4143	0.4490
8	−0.3243	0.1597	0.4841	0.1322	−0.4419	0.5741
9	−0.3243	0.1597	0.4840	0.1322	−0.4418	0.5740
10	−0.3490	0.1791	0.5281	0.1065	−0.3701	0.4766
11	−0.3670	0.1791	0.5461	0.1065	−0.3701	0.4766
12	−0.3490	0.1597	0.5087	0.1065	−0.4419	0.5483
13	−0.3490	0.1597	0.5086	0.1065	−0.4418	0.5483
14	−0.3670	0.1597	0.5267	0.1065	−0.4419	0.5484
15	−0.3670	0.1597	0.5267	0.1065	−0.4418	0.5484

(25–100 °C) with Lewis acid catalysis or at high pressure.[16,17] Gas-phase reaction 3 at 1 atm and 230 °C yielded 8% of the adduct but the reaction in acetone at 14 kbar and 80 °C gave 35% adduct and 60% dimerization product of acrolein.[18] The relative energy of the LUMO of the 1-oxabutadiene systems estimated by one-electron half-wave reduction potentials ($E_{1/2}^{\text{red}}$) showed good correlations with kinetic results and confirmed that the reactions of 1-oxabutadienes are reverse electron demand reactions under FMO control.[19]

Reaction of acrolein and formaldehyde: reaction 4 ($O{=}CH{-}CH{=}CH_2 + CH_2{=}O$). The two forming bonds are between O_1 and C_5 (d_5) and C_4 and O_6 (d_6). Bond formation between the two heteroatoms (O) seems to be unfavorable energetically due to the lower bond energy of O–O (35 kcal mol^{-1}) compared to C–O (85 kcal mol^{-1}) and to oxygen atom lone pair–lone pair repulsions.[19] The formation of the d_6 bond (44%) has progressed slightly more in the transition structure than d_5 (42%) with $\Delta d = d_5 - d_6 = 0.027$ Å. This reaction is a normal electron demand type, chiefly because of a much greater lowering of the dienophile HOMO ($\Delta\varepsilon = \varepsilon_{\text{ethylene}} - \varepsilon_{\text{formaldehyde}} = 0.155$ hartree) than the diene HOMO level ($\Delta\varepsilon = 0.062$ hartree). For this reaction the DFT activation energy is lower (by 3–5 kcal mol^{-1}) than that for reactions 1–3, in contrast to the reaction energy (ΔE°), which is higher by more than 10 kcal mol^{-1}. The transition structure of this reaction is characterized by relatively short forming bonds and hence the TS is relatively late compared to those for reactions 1–3 as indicated by the greater degree of progress in bond formation. Thus the lower activation energy obtained for reaction 4 is a reflection of the greater degree of bond formation in the TS between O and C (bond energy ≅ 85 kcal mol^{-1}) than between C and C (bond energy ≅ 83 kcal mol^{-1}) in reactions 1–3. Thus the energy stabilization gained by the greater degree of bond formation between O and C in the TS seems to more than compensate for the loss incurred by the lower thermodynamic driving force. It should be noted that this reaction is the only example where the MP2 $\Delta E^{\neq}$ value is significantly higher than the DFT value. It is notable that the reaction of acrolein (O in diene) leads to a reverse electron demand interaction, whereas the reaction of formaldehyde (O in dienophile) leads to a normal electron demand reaction. When the oxygen heteroatom is present in both reactants (reaction 4), the overall electron demand is in favor of the heteroatom in the dienophile, that is the effect of the heterodienophile prevails over that of the heterodiene.

Reactions involving dienophile and/or diene with a terminal sulfur heteroatom

Reactions 5–7 ($CH_2{=}CH{-}CH{=}CH_2 + CH_2{=}S$, $S{=}CH{-}CH{=}CH_2 + CH_2{=}CH_2$, $S{=}CH{-}CH{=}CH_2 + CH_2{=}S$). The transition structure of reaction 5 is highly asynchronous with a longer bond between C_4 and S_6 ($d_6 = 2.652$ Å) than between the two carbons ($d_5 = 2.293$ Å). This large difference in the two forming bonds ($\Delta d = -0.359$ Å) results from the longer covalent radius of the second row element S than the first row element C and the larger LUMO coefficient of C_5 than S_6. As can be seen in Table 4, this reaction is a normal electron demand type with $\Delta\varepsilon_{\text{FMO}} = 0.378$ hartree, which is the narrowest inter-frontier level gap observed in this work. As can be expected from a narrow inter-frontier level gap, the activation barrier is much lower than those for the reactions involving an oxygen heteroatom, that is the $\Delta E^{\neq}$ value for reaction 5 is lower by as much as ~15 kcal mol^{-1} than that for reaction 2. This reaction is also facilitated by a greater thermodynamic driving force than the corresponding oxygen analog ($-\delta\Delta E^{\circ} = 14$ kcal mol^{-1}).

Due to similar reasons given above for reaction 5, the transition structure of the reaction between 1-thiabutadiene and ethylene, reaction 6, is relatively loose and highly asynchronous with d_5 (2.626 Å) longer than d_6 (2.257 Å). This reaction is a reverse electron demand type with a lower $\Delta\varepsilon_{\text{FMO}}$ for a diene LUMO–dienophile HOMO interaction ($\Delta\varepsilon_{\text{FMO}} = 0.405$ *vs.* 0.528 hartree) and a larger LUMO coefficient for C_4 than S_1. The relatively narrow ($\Delta\varepsilon_{\text{FMO}}$ is greater by only 0.027 hartree than that for reaction 5) inter-frontier level gap is again a favorable factor, as is the large exothermicity (ΔE° is more negative by 12 kcal mol^{-1} than in the corresponding oxygen analog, reaction 3). It is, however, notable that those two favorable factors, the narrow $\Delta\varepsilon_{\text{FMO}}$ and low ΔE°, are somewhat less so than those for reaction 5. Thus the activation energy is lower than that for the corresponding oxygen analog, $\delta\Delta E^{\neq} = \Delta E^{\neq}_{\text{rxn}\,3} - \Delta E^{\neq}_{\text{rxn}\,6} \cong 13$ kcal mol^{-1}, but the difference, $\delta\Delta E^{\neq}$, is less than that for reaction 5 ($\delta\Delta E^{\neq} \cong 15$ kcal mol^{-1}).

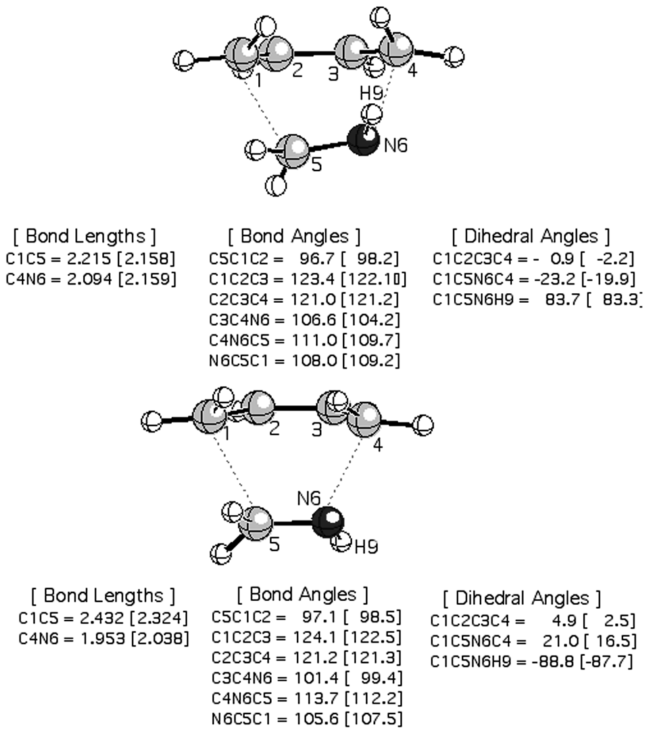
The reaction of 1-thiabutadiene and thioformaldehyde (reaction 7) has the lowest energy transition structure with a longer forming S_1–C_5 ($d_5 = 2.848$ Å) bond than the forming C_4–S_6 ($d_6 = 2.802$ Å) bond. Here again bond formation between the two S atoms is unfavorable to lone pair–lone pair repulsions. The two forming bonds are much longer than those in other transition structures, but the difference in the two is small, d_5 (18%) *vs.* d_6 (20%), so that a near synchronous structure is obtained. This reaction is a normal electron demand type but the two relatively small $\Delta\varepsilon_{\text{FMO}}$ values differ little with $\Delta\varepsilon_{\text{FMO}} = 0.402$ and 0.449 hartree for the normal and reverse electron demand interactions, respectively. This reaction has the lowest activation energy, $\Delta E^{\neq}$(DFT) = −0.1 kcal mol^{-1} and $\Delta E^{\neq}$(MP2) = −0.4 kcal mol^{-1} of all the reactions studied. Compared to the corresponding reaction of the oxygen analog, reaction 4, the $\Delta E^{\neq}$ value is lower by 18 kcal mol^{-1}, which is almost entirely ascribable to the stronger thermodynamic driving force, $\delta\Delta E^0 = \Delta E^0_{\text{s}} - \Delta E^0_{\text{O}} \cong -20$ kcal mol^{-1}. We note here again that the reaction involving the heteroatom S in both diene and dienophile proceeds by a normal electron demand interaction so that the effect of the heterodienophile prevails over that of the heterodiene in determining the FMO interaction pattern, as it was found for the reactions involving the heteroatom O.

In general, the hetero-Diels–Alder reactions of thioformaldehyde and 1-azabutadiene are highly exothermic and have loose and relatively early transition states due to the longer forming bond between a second row element (S) and carbon, which results in much reduced deformation energies and interhydrogen steric interactions leading to much lower activation barriers. The decrease in the deformation energies and steric effects more than compensate for the unfavorable energy loss due to the weaker bond strength of the C–S (65 kcal mol^{-1}) bond than the C–C (83 kcal mol^{-1}) and C–O (85 kcal mol^{-1}) bonds.

Experimentally, thiocarbonyl compounds are found to be more reactive, versatile dienophiles than are the corresponding carbonyl compounds.[20] It is known that 1-thiabutadienes are electron deficient and react with electron-rich or strained dienophiles in reverse electron demand Diels–Alder reactions. However, normal electron demand reactions are also widely investigated with electron-deficient dienophiles;[20] in such cases, the complementary addition of electron-donating substituents to C_2 and C_4 of the 1-thiabutadiene system enhances both the rate and regioselectivity of the normal electron demand reactions.

Reactions involving dienophile and/or diene with a terminal nitrogen heteroatom

Reaction of butadiene and formaldimine: reactions 8 and 9 ($CH_2{=}CH{-}CH{=}CH_2 + CH_2{=}NH$). For this reaction, two transition structures were found with the imino hydrogen *exo* (reaction 8) and *endo* (reaction 9) as shown in Fig. 1. In both



[Bond Lengths]
C1C5 = 2.215 [2.158]
C4N6 = 2.094 [2.159]

[Bond Angles]
C5C1C2 = 96.7 [98.2]
C1C2C3 = 123.4 [122.10]
C2C3C4 = 121.0 [121.2]
C3C4N6 = 106.6 [104.2]
C4N6C5 = 111.0 [109.7]
N6C5C1 = 108.0 [109.2]

[Dihedral Angles]
C1C2C3C4 = - 0.9 [-2.2]
C1C5N6C4 = -23.2 [-19.9]
C1C5N6H9 = 83.7 [83.3]

[Bond Lengths]
C1C5 = 2.432 [2.324]
C4N6 = 1.953 [2.038]

[Bond Angles]
C5C1C2 = 97.1 [98.5]
C1C2C3 = 124.1 [122.5]
C2C3C4 = 121.2 [121.3]
C3C4N6 = 101.4 [99.4]
C4N6C5 = 113.7 [112.2]
N6C5C1 = 105.6 [107.5]

[Dihedral Angles]
C1C2C3C4 = 4.9 [2.5]
C1C5N6C4 = 21.0 [16.5]
C1C5N6H9 = -88.8 [-87.7]

Fig. 1 DFT (B3LYP/6-31G*) and MP2 (in brackets) results for the transition structures for reactions 8 (top) and 9 (bottom). Bond lengths are in Å and angles are in degrees.

the *exo* and *endo* transition structures, the forming C_4–N_6 (d_6) bond is shorter than the C_1–C_5 (d_5) bond by 0.121 and 0.479 Å with MP2 and DFT, respectively. The *endo* TS is considerably more asynchronous (23% d_5 and 45% d_6) than the *exo* form (32% d_5 and 35% d_6). In fact the *exo* form has an almost synchronous transition structure as the two bond lengths calculated with MP2 ($d_5 = 2.158$ and $d_6 = 2.159$ Å) indicate. Here again the DFT transition structure is much more asynchronous than the MP2 structure. The greater extent of bond formation for C–N than C–C in both of the diastereomeric transition structures suggests that for the reaction of formaldimine FMO theory does not apply. This is because normally a greater LUMO coefficient is expected on the opposite terminal carbon atom (C_5) in a heterodienophile (and heterodiene as well) with a terminal heteroatom, and in contrast to the present results FMO theory predicts that the C_1–C_5 bond should be formed more fully than the C_4–N_6 bond in the transition structure. Moreover, this reaction proceeds by a normal electron demand interaction (Table 4) so that the dienophile LUMO interacts with the diene HOMO in the TS.[5a,16b] ($\Delta\varepsilon_{\mathrm{FMO}} = 0.484$ and 0.574 hartree for the normal and reverse electron demand interactions, respectively.) The NPA also show clearly that the direction of electron donation in the transition structure is from butadiene to the imine.

We can suggest two reasons for this anomalous behavior of the formaldimine. The two terminal atoms, C_5 [LUMO coefficients (MP2) for $2p_z$ and $3p_z$ are 0.373 and 0.721] and N_6 [LUMO coefficients (MP2) for $2p_z$ and $3p_z$ are -0.380 and -0.628] have similar LUMO coefficients and the steric effects between two hydrogens on C_1 (H_1 and H_2) and on C_5 (H_7 and H_8) inhibit bond formation between C_1 and C_5. Close examination of the two transition structures show significant twisting between butadiene and formaldimine. To reduce repulsive interaction of the nitrogen lone pair (σ type) with the butadiene π system, the lone pair is turned away from the butadiene π system by twisting the dienophile. In the *exo* form (*exo* H with *endo* lone pair) the lone pair is turned downward anticlockwise by 19.9° (dihedral angle C_1–C_5–N_6–$C_4 = -19.9°$) and in the *endo* form (*endo* H with *exo* lone pair) the lone pair is turned downward clockwise by 16.5°. As a result of this dienophile twisting, terminal hydrogen atoms (H_7 and H_8) on C_5 sterically interact with those (H_1 and H_2) on C_1; the steric repulsions between those hydrogens are counterbalanced by the similar steric effects between hydrogen atoms on C_4 (H_5 and H_6) and N_6 (H_9) plus the lone pair–π orbital repulsion.[3d] The inter-hydrogen distances are collected in Table 5. Closer approach of the N_6 atom toward C_4 in the *endo* than in the *exo* transition structure is reflected in the closer inter-hydrogen distances (d_{27} is 2.264 and 2.477 Å in the *endo* and *exo* forms, respectively) in the *endo* form. Actually d_{27} is shorter than d_5 (2.432 Å) in the *endo* TS.

The activation barrier to the *endo* approach is lower by *ca.* 4 kcal mol^{-1} than that of the *exo* approach. The *endo* imino hydrogen preference, $\delta\Delta E^{\neq}(\Delta E^{\neq}_{\mathrm{endo}} - \Delta E^{\neq}_{\mathrm{exo}} < 0)$, seems to depend more on the basis set level used than on the level to which electron correlation effects are accounted for, albeit the absolute magnitude of the activation energies are lowered. For example, the $\delta\Delta E^{\neq}$ values[3d] are 5.3 (3-21G//3-21G), 4.3 (6-31G*//3-21G*), 4.9 (MP2/6-31G*//3-21G*), 3.9 (MP2/6-31G*//MP2/6-31G*), 3.9 (MP3/6-31G*//MP2/6-31G*) and 3.9 (MP4SDTQ/6-31G*//MP2/6-31G*) kcal mol^{-1}. Our MP2 (4.0 kcal mol^{-1}) and DFT (3.8 kcal mol^{-1}) values are in close agreement with that at the highest level of theory, MP4/6-31G*. Since the exothermicity is the same in both reactions (-29.3 kcal mol^{-1}) the *endo* preference of ~4 kcal mol^{-1} must be of purely kinetic origin. In the *exo* transition structure, there will be a greater lone pair–π system repulsion since the lone pair on N is in the *endo* direction, which should require a greater twisting of the *exo* form ($-19.9°$ *vs.* 16.5° for exo *vs.* endo) to alleviate the repulsive interactions. The greater amount of twisting in the *exo* transition structure not only raises repulsive energy by a greater amount but also

Table 5 The distances ($d_{\mathrm{H,H}}$ in Å) between hydrogen atoms in the transition structures for the reactions of butadiene with formaldimine (reactions 8 and 9) and of 1-azabutadiene with ethylene (reactions 10 and 11) (See Scheme 1 for the atom numbering scheme.)

Butadiene + formaldimine			1-Azabutadiene + ethylene		
$d_{\mathrm{H,H}}$	*endo*	*exo*	$d_{\mathrm{H,H}}$	*endo*	*exo*
d_{17}	3.158	2.722	d_{27}	2.294	2.791
d_{18}	2.929	2.952	d_{28}	2.862	2.551
d_{27}	2.264	2.477	d_{37}	4.229	4.326
d_{28}	3.021	3.112	d_{38}	3.262	3.426
d_{37}	4.380	4.087	d_{59}	3.072	3.090
d_{38}	3.639	3.107	d_{69}	2.656	2.753
d_{49}	2.921	4.292	d_{510}	2.357	2.389
d_{59}	3.000	2.173	d_{610}	2.607	2.591
d_{67}	2.921	4.292	d_{49}	3.241	3.283
			d_{410}	4.178	4.188

increases the inter-hydrogen steric repulsion. We note in Table 5 that d_{59} (2.173 Å) is very short in the *exo* transition structure.

Various types of imine dienophiles, such as *N*-sulfonylimines,[21a] *N*-acylimines,[20a] cyclic imines[21b] and *C*-acylimines[21c,d] are used in the synthetic Diels–Alder reactions. Experimentally, imine dienophiles are known to react by normal electron demand interactions.[5a,16b] The TS was reported to be concerted but asynchronous and exhibits *endo* preference of the imino hydrogen.[21e,f]

Reaction of 1-azabutadiene and ethylene: reactions 10 and 11 (NH=CH–CH=CH$_2$ + CH$_2$=CH$_2$). A terminal nitrogen heteroatom in the diene lowers the LUMO level sufficiently to make the reaction a reverse electron demanding one ($\Delta\varepsilon_{\mathrm{FMO}} = 0.477$ hartree). Experimentally, the electrophilic character of 1-azadienes is well recognized, which leads to the reverse electron demand Diels–Alder reaction.[22] In both the *exo* and *endo* transition structures, the C_4–C_6 (d_6) bond is shorter than the C_1–N_5 (d_5) bond as expected based on FMO theory. The *endo* form is more asynchronous (25% d_5 and 42% d_6) than the *exo* form (29% d_5 and 35% d_6), which is closer to a synchronous structure, Fig. 2. In these reactions, the activation energy is higher for the *endo* form by *ca.* 4 kcal mol^{-1} than the *exo* form, which is in contrast to a lower barrier to the *endo* approach by a similar amount in the reactions of butadiene and formaldimine. Since here again the reaction energies ($\Delta E°$) through the *exo* and *endo* transition structures are similar (-33 kcal mol^{-1}), the *exo* preference should have a kinetic origin. The inter-hydrogen distances in the *endo* transition structure are relatively shorter than those in the *exo* structure so that the inter-hydrogen steric effects can be a factor in favor of the *exo* approach (Table 5). Another factor contributing to the *exo* preference could be the larger HOMO coefficients of the terminal atoms (N_1 and C_4) for the *exo* diene: the MP2 LUMO coefficients of the N_1 atom are 0.289, -0.245 (*exo*) and -0.273, -0.231 (*endo*) and those of the C_4 atom are 0.329, 0.267 (*exo*) and 0.314, 0.264 (*endo*) for $2p_z$ and $3p_z$, respectively.

We can conclude that a terminal heteroatom, O, S or N, lowers the LUMO of the diene sufficiently, leading to a reverse electron demand type reaction. A terminal nitrogen on the dienophile favors the *endo* whereas that on the diene favors the *exo* transition structure by about the same amount (4 kcal mol^{-1}) and both are strongly influenced by inter-terminal-hydrogen steric effects.

Reaction of 1-azabutadiene and formaldimine: reactions 12–15 (NH=CH–CH=CH$_2$ + CH$_2$=NH). There are four combinations, depending on the stereochemistry of the two imino hydrogens, for reactions 12–15; these are *exo–exo* (*exo* diene–*exo* dienophile), *exo–endo*, *endo–exo* and *endo–endo*. The effects of the nitogen heteroatom in the dienophile prevail over those in the diene and all 4 reactions are the normal electron demand type with a shorter forming C_4–N_6 (d_6) bond than forming N_1–C_4 (d_5) bond. The transition structure becomes successively more asynchronous as we proceed from reaction 12 (29% d_5 and 43% d_6) to 15 (20% d_5 and 57% d_6). The transition structures are shown in Fig. 3 and 4. Bond formation between the two nitrogen atoms is again unfavorable, as was found with the other heteroatoms, O (reaction 4) and S (reaction 7); bond energies are N–N (39 kcal mol^{-1}) and C–N (73 kcal mol^{-1}).

The *exo* preference of the diene imino hydrogen and the *endo* preference of the dienophile imino hydrogen are reflected in the respective activation energies in reactions 12–15. For these reactions the reaction energies are nearly constant, $\Delta E° = -24$ kcal mol^{-1}, which is higher by 5 and 9 kcal mol^{-1} than those for the reactions of formaldimine with butadiene (reactions 8 and 9) and 1-azabutadiene with ethylene (reactions 10 and 11), respectively. For each preferred stereochemistry, there is an activation energy lowering of 5–6 kcal

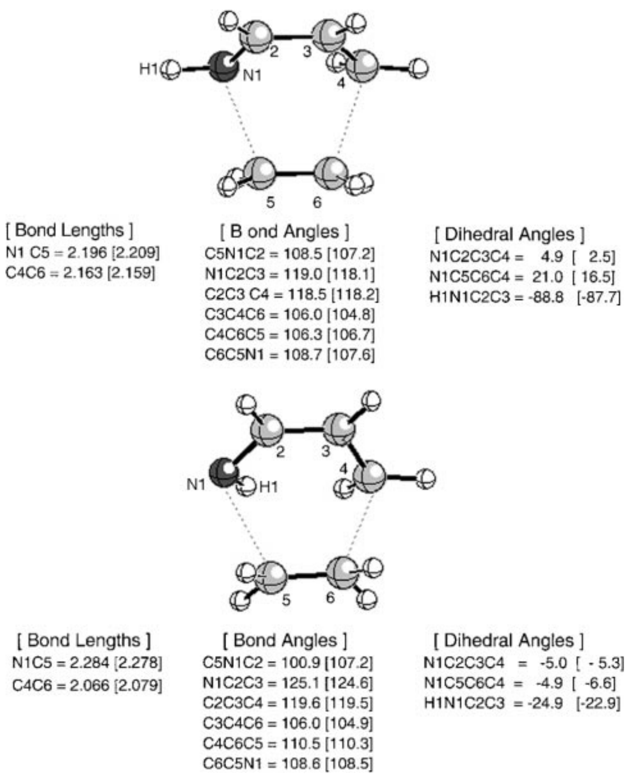


Fig. 2 DFT (B3LYP/6-31G*) and MP2 (in brackets) results for the transition structures for reactions 10 (top) and 11 (bottom). Bond lengths are in Å and angles are in degrees.

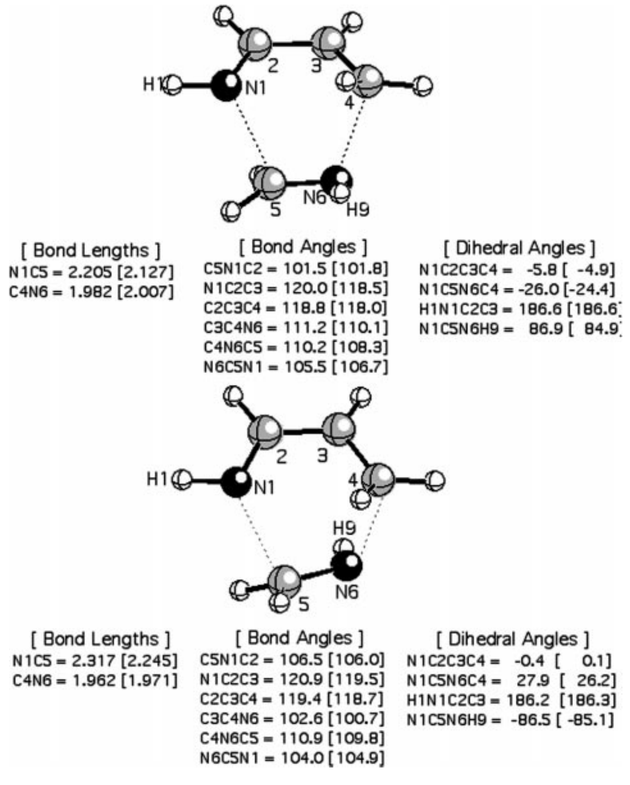


Fig. 3 DFT (B3LYP/6-31G*) and MP2 (in brackets) results for the transition structures for reactions 12 (top) and 13 (bottom). Bond lengths are in Å and angles are in degrees.

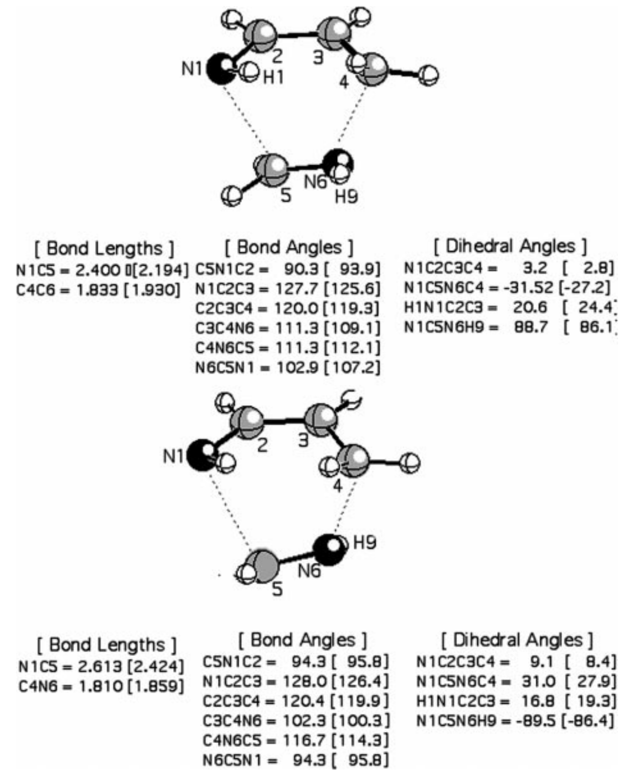


Fig. 4 DFT (B3LYP/6-31G*) and MP2 (in brackets) results for the transition structures for reactions 14 (top) and 15 (bottom). Bond lengths are in Å and angles are in degrees.

mol^{-1} ($\delta\Delta E^{\neq} = \Delta E^{\neq}_{(exo-exo)} - \Delta E^{\neq}_{(exo-endo)} \cong 5$ kcal mol^{-1}, $\Delta E^{\neq}_{(endo-exo)} - \Delta E^{\neq}_{(endo-endo)} \cong 6$ kcal mol^{-1}), which is 1–2 kcal mol^{-1} higher than that for reactions with a terminal N heteroatom in a single reactant (*vide supra*). The higher values of 5–6 kcal mol^{-1} in the double heteroatom reactions than the nearly constant preference energy of 4 kcal mol^{-1} in the single heteroatom reactions are partially due to the higher reaction energies for the double heteroatom reactions (by 5–9 kcal mol^{-1}) than for the single heteroatom reactions.

Summary

The results of this work can be summarized as follows.

(*i*) All transition structures are located at a relatively early position along the reaction coordinate and have concerted and asynchronous structures. The DFT calculations lead to more asynchronous transition structures than the MP2 results.

(*ii*) The reactions of formaldehyde, thioformaldehyde and formaldimine with butadiene and heterodienes having the same heteroatom proceed by diene HOMO–dienophile LUMO (normal electron demand) interactions whereas those of ethylene with acrolein, 1-thiabutadiene and 1-azabutadiene proceed by diene LUMO–dienophile HOMO (reverse electron demand) interactions. This latter reverse electron demand is caused by the lowering of the LUMO of the heterodiene.

(*iii*) The C–C bond is more fully formed than the heteroatom–carbon bond in all the transition structures with the exception of the reaction between formaldimine and butadiene for which C–N bond formation is ahead of C–C bond making.

(*iv*) All the reactions are highly exothermic.

(*v*) The reactions are faciliated by heteroatoms in both the diene and dienophile, but bond formation between two heteroatoms is disfavored.

(*vi*) There is an *endo* preference of the imino hydrogen for formaldimine but an *exo* preference is exhibited by that of 1-azabutadiene. The stereoselective preference is *ca.* 4 kcal mol^{-1} in both cases, which is mostly due to inter-hydrogen steric effects.

(*vii*) Overall, the DFT calculations of the hetero-Diels–Alder reactions provide an economical way of accounting for electron correlation effects at nearly the same level as the MP2 ones.

Acknowledgements

We are grateful to the Ministry of Education and Inha University for support of this work.

Notes and references

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